**What You’ll Learn**

- Translate verbal sentences into equations.
- Translate equations into verbal sentences.

**Vocabulary**

- four-step problem-solving plan
- defining a variable formula

**How are equations used to describe heights?**

The Statue of Liberty sits on a pedestal that is 154 feet high. The height of the pedestal and the statue is 305 feet. If \( s \) represents the height of the statue, then the following equation represents the situation.

\[ 154 + s = 305 \]

**WRITE EQUATIONS** When writing equations, use variables to represent the unspecified numbers or measures referred to in the sentence or problem. Then write the verbal expressions as algebraic expressions. Some verbal expressions that suggest the equals sign are listed below:

- is
- equals
- is equal to
- is the same as
- is as much as
- is identical to

**Example 1 Translate Sentences into Equations**

Translate each sentence into an equation.

**a.** Five times the number \( a \) is equal to three times the sum of \( b \) and \( c \).

\[
\frac{5}{3} \times \frac{a}{b} = \frac{3}{3} \times \frac{(b + c)}{(b + c)}
\]

The equation is \( 5a = 3(b + c) \).

**b.** Nine times \( y \) subtracted from 95 equals 37.

Rewrite the sentence so it is easier to translate.

\[
\frac{95}{95} - \frac{9y}{9y} = \frac{37}{37}
\]

The equation is \( 95 - 9y = 37 \).
Using the four-step problem-solving plan can help you solve any word problem.

**Key Concept**

**Four-Step Problem-Solving Plan**

**Step 1** Explore the problem.

**Step 2** Plan the solution.

**Step 3** Solve the problem.

**Step 4** Examine the solution.

Each step of the plan is important.

**Step 1** **Explore the Problem**
To solve a verbal problem, first read the problem carefully and explore what the problem is about.
- Identify what information is given.
- Identify what you are asked to find.

**Step 2** **Plan the Solution**
One strategy you can use to solve a problem is to write an equation. Choose a variable to represent one of the unspecified numbers in the problem. This is called **defining a variable**. Then use the variable to write expressions for the other unspecified numbers in the problem. You will learn to use other strategies throughout this book.

**Step 3** **Solve the Problem**
Use the strategy you chose in Step 2 to solve the problem.

**Step 4** **Examine the Solution**
Check your answer in the context of the original problem.
- Does your answer make sense?
- Does it fit the information in the problem?

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**Example 2** **Use the Four-Step Plan**

**ICE CREAM** Use the information at the left. In how many days can 40,000,000 gallons of ice cream be produced in the United States?

**Explore**
You know that 2,000,000 gallons of ice cream are produced in the United States each day. You want to know how many days it will take to produce 40,000,000 gallons of ice cream.

**Plan**
Write an equation to represent the situation. Let \( d \) represent the number of days needed to produce the ice cream.

\[
\frac{2,000,000}{2,000,000} \times \frac{\text{times}}{d} = \frac{40,000,000}{d}
\]

**Solve**
\[
2,000,000d = 40,000,000 \quad \text{Find } d \text{ mentally by asking, "What number times } 2,000,000 \text{ equals } 40,000,000?" \]

\[
d = 20
\]

It will take 20 days to produce 40,000,000 gallons of ice cream.

**Examine**
If 2,000,000 gallons of ice cream are produced in one day, \( 2,000,000 \times 20 \) or 40,000,000 gallons are produced in 20 days. The answer makes sense.
A formula is an equation that states a rule for the relationship between certain quantities. Sometimes you can develop a formula by making a model.

**Algebra Activity**

**Surface Area**

- Mark each side of a rectangular box as the length \( \ell \), the width \( w \), or the height \( h \).
- Use scissors to cut the box so that each surface or face of the box is a separate piece.

**Analyze**

1. Write an expression for the area of the front of the box.
2. Write an expression for the area of the back of the box.
3. Write an expression for the area of one side of the box.
4. Write an expression for the area of the other side of the box.
5. Write an expression for the area of the top of the box.
6. Write an expression for the area of the bottom of the box.
7. The surface area of a rectangular box is the sum of all the areas of the faces of the box. If \( S \) represents surface area, write a formula for the surface area of a rectangular box.

**Make a Conjecture**

8. If \( s \) represents the length of the side of a cube, write a formula for the surface area of a cube.

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**Example 3 Write a Formula**

Translate the sentence into a formula.

The perimeter of a rectangle equals two times the length plus two times the width.

**Words**

Perimeter equals two times the length plus two times the width.

**Variables**

Let \( P = \text{perimeter} \), \( \ell = \text{length} \), and \( w = \text{width} \).

**Formula**

\[
P = 2\ell + 2w
\]

The formula for the perimeter of a rectangle is \( P = 2\ell + 2w \).

**WRITE VERBAL SENTENCES** You can also translate equations into verbal sentences or make up your own verbal problem if you are given an equation.

**Example 4 Translate Equations into Sentences**

Translate each equation into a verbal sentence.

a. \( 3m + 5 = 14 \)

\[
3m + 5 = 14
\]

Three times \( m \) plus five equals fourteen.
b. \( w + v = y^2 \)

\[
\begin{align*}
\frac{w + v}{\phantom{y^2}} &= \phantom{\frac{y}{y}} y^2 \\
The \text{ sum of } w \text{ and } v \text{ equals } \text{ the square of } y.
\end{align*}
\]

**Example 5** Write a Problem

Write a problem based on the given information.

\[ a = \text{Rafael's age} \quad a + 5 = \text{Tierra's age} \quad a + 2(a + 5) = 46 \]

You know that \( a \) represents Rafael’s age and \( a + 5 \) represents Tierra’s age. The equation adds \( a \) plus twice \((a + 5)\) to get 46. A sample problem is given below.

Tierra is 5 years older than Rafael. The sum of Rafael’s age and twice Tierra’s age equals 46. How old is Rafael?

**Check for Understanding**

**Concept Check**

1. List the four steps used in solving problems.
2. Analyze the following problem.
   Misae has $1900 in the bank. She wishes to increase her account to a total of $3500 by depositing $30 per week from her paycheck. Will she reach her savings goal in one year?
   a. How much money did Misae have in her account at the beginning?
   b. How much money will Misae add to her account in 10 weeks? in 20 weeks?
   c. Write an expression representing the amount added to the account after \( w \) weeks have passed.
   d. What is the answer to the question? Explain.
3. **OPEN ENDED** Write a problem that can be answered by solving \( x + 16 = 30 \).

**Guided Practice**

Translate each sentence into an equation.

4. Two times a number \( t \) decreased by eight equals seventy.
5. Five times the sum of \( m \) and \( n \) is the same as seven times \( n \).

Translate each sentence into a formula.

6. The area \( A \) of a triangle equals one half times the base \( b \) times the height \( h \).
7. The circumference \( C \) of a circle equals the product of two, pi, and the radius \( r \).

Translate each equation into a verbal sentence.

8. \( 14 + d = 6d \)
9. \( \frac{1}{3}b - \frac{3}{4} = 2a \)

10. Write a problem based on the given information.
    \[ c = \text{cost of a suit} \quad c - 25 = 150 \]

**Application** WRESTLING For Exercises 11 and 12, use the following information.

Darius is training to prepare for wrestling season. He weighs 155 pounds now. He wants to gain weight so that he starts the season weighing 160 pounds.

11. If \( g \) represents the number of pounds he wants to gain, write an equation to represent the situation.

12. How many pounds does Darius need to gain to reach his goal?
Translate each sentence into an equation.

13. Two hundred minus three times $x$ is equal to nine.
14. The sum of twice $r$ and three times $s$ is identical to thirteen.
15. The sum of one-third $q$ and 25 is as much as twice $q$.
16. The square of $m$ minus the cube of $n$ is sixteen.
17. Two times the sum of $v$ and $w$ is equal to two times $z$.
18. Half of the sum of nine and $p$ is the same as $p$ minus three.
19. The number $g$ divided by the number $h$ is the same as seven more than twice the sum of $g$ and $h$.
20. Five-ninths the square of the sum of $a$, $b$, and $c$ equals the sum of the square of $a$ and the square of $c$.

21. GEOGRAPHY  The Pacific Ocean covers about 46% of Earth. If $P$ represents the area of the Pacific Ocean and $E$ represents the area of Earth, write an equation for this situation.

22. GARDENING  Mrs. Patton is planning to place a fence around her vegetable garden. The fencing costs $1.75 per yard. She buys $f$ yards of fencing and pays $3.50 in tax. If the total cost of the fencing is $73.50, write an equation to represent the situation.

Translate each sentence into a formula.

23. The area $A$ of a parallelogram is the base $b$ times the height $h$.
24. The volume $V$ of a pyramid is one-third times the product of the area of the base $B$ and its height $h$.

25. The perimeter $P$ of a parallelogram is twice the sum of the lengths of the two adjacent sides, $a$ and $b$.
26. The volume $V$ of a cylinder equals the product of $\pi$, the square of the radius $r$ of the base, and the height.

27. In a right triangle, the square of the measure of the hypotenuse $c$ is equal to the sum of the squares of the measures of the legs, $a$ and $b$.
28. The temperature in degrees Fahrenheit $F$ is the same as nine-fifths of the degrees Celsius $C$ plus thirty-two.
Translate each equation into a verbal sentence.

29. \( d - 14 = 5 \)  
30. \( 2f + 6 = 19 \)  
31. \( k^2 + 17 = 53 - j \)

32. \( 2a = 7a - b \)  
33. \( \frac{3}{4}p + \frac{1}{2} = p \)  
34. \( \frac{2}{5}w = \frac{1}{2}w + 3 \)

35. \( 7(m + n) = 10n + 17 \)  
36. \( 4(t - s) = 5s + 12 \)

37. **GEOMETRY** If \( a \) and \( b \) represent the lengths of the bases of a trapezoid and \( h \) represents its height, then the formula for the area \( A \) of the trapezoid is \( A = \frac{1}{2}h(a + b) \). Write the formula in words.

38. **SCIENCE** If \( r \) represents rate, \( t \) represents time, and \( d \) represents distance, then \( rt = d \). Write the formula in words.

Write a problem based on the given information.

39. \( y = \) Yolanda's height in inches  
\( y + 7 = \) Lindsey's height in inches  
\( 2y + (y + 7) = 193 \)

40. \( p = \) price of a new backpack  
\( 0.055p = \) tax  
\( p + 0.055p = 31.65 \)

**GEOMETRY** For Exercises 41 and 42, use the following information.
The volume \( V \) of a cone equals one-third times the product of \( \pi \), the square of the radius \( r \) of the base, and the height \( h \).

41. Write the formula for the volume of a cone.
42. Find the volume of a cone if \( r \) is 10 centimeters and \( h \) is 30 centimeters.

**GEOMETRY** For Exercises 43 and 44, use the following information.
The volume \( V \) of a sphere is four-thirds times \( \pi \) times the radius \( r \) of the sphere cubed.

43. Write a formula for the volume of a sphere.
44. Find the volume of a sphere if \( r \) is 4 inches.

**LITERATURE** For Exercises 45–47, use the following information.
Edgar Rice Burroughs is the author of the *Tarzan of the Apes* stories. He published his first Tarzan story in 1912. Some years later, the town in southern California where he lived was named Tarzana.

45. Let \( y \) represent the number of years after 1912 that the town was named Tarzana. Write an expression for the year the town was named.
46. The town was named in 1928. Write an equation to represent the situation.
47. Use what you know about numbers to determine the number of years between the first Tarzan story and the naming of the town.

**TELEVISION** For Exercises 48–51, use the following information.
During a highly rated one-hour television program, the entertainment portion lasted 15 minutes longer than 4 times the advertising portion.

48. If \( a \) represents the time spent on advertising, write an expression for the entertainment portion.
49. Write an equation to represent the situation.
50. Use your equation and the guess-and-check strategy to determine the number of minutes spent on advertising. Choose different values of \( a \) and evaluate to find the solution.
51. Time the entertainment and advertising portions of a one-hour television program you like to watch. Describe what you found. Are the results of this problem similar to your findings?
52. **CRITICAL THINKING**  The surface area of a prism is the sum of the areas of the faces of the prism. Write a formula for the surface area of the triangular prism at the right.

53. **WRITING IN MATH**  Answer the question that was posed at the beginning of the lesson.

How are equations used to describe heights?
Include the following in your answer:
- an equation relating the Sears Tower, which is 1454 feet tall; the twin antenna towers on top of the building, which are \(a\) feet tall; and a total height, which is 1707 feet, and
- an equation representing the height of a building of your choice.

54. Which equation represents the following sentence?
   *One fourth of a number plus five equals the number minus seven.*
   - **A** \(\frac{1}{4}n + 7 = n - 5\)
   - **B** \(\frac{1}{4}n + 5 = n - 7\)
   - **C** \(4n + 7 = n - 5\)
   - **D** \(4n + 5 = n - 7\)

55. Which sentence can be represented by \(7(x + y) = 35\)?
   - **A** Seven times \(x\) plus \(y\) equals 35.
   - **B** One seventh of the sum of \(x\) and \(y\) equals 35.
   - **C** Seven plus \(x\) and \(y\) equals 35.
   - **D** Seven times the sum of \(x\) and \(y\) equals 35.

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**Standardized Test Practice**

56. \(\sqrt{8100}\)
57. \(-\sqrt{\frac{25}{36}}\)
58. \(\sqrt{90}\)
59. \(-\sqrt{55}\)

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57. \(-\sqrt{\frac{25}{36}}\)  
58. \(\sqrt{90}\)  
59. \(-\sqrt{55}\)

**Maintain Your Skills**

**Mixed Review**

Find each square root. Use a calculator if necessary. Round to the nearest hundredth if the result is not a whole number or a simple fraction.  *(Lesson 2-7)*

56. \(\sqrt{8100}\)  
57. \(-\sqrt{\frac{25}{36}}\)  
58. \(\sqrt{90}\)  
59. \(-\sqrt{55}\)

Find the probability of each outcome if a die is rolled.  *(Lesson 2-6)*

60. a 6  
61. an even number  
62. a number greater than 2

Simplify each expression.  *(Lesson 1-5)*

63. \(12d + 3 - 4d\)  
64. \(7t^2 + t + 8t\)  
65. \(3(a + 2b) + 5a\)

Evaluate each expression.  *(Lesson 1-2)*

66. \(5(8 - 3) + 7 \cdot 2\)  
67. \(6(4^3 + 2^2)\)  
68. \(7(0.2 + 0.5) - 0.6\)

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**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL**  Find each sum or difference.  *(To review operations with fractions, see pages 798 and 799.)*

69. \(5.67 + 3.7\)  
70. \(0.57 + 2.8\)  
71. \(5.28 - 3.4\)  
72. \(9 - 7.35\)

73. \(\frac{2}{3} + \frac{1}{5}\)  
74. \(\frac{1}{6} + \frac{2}{3}\)  
75. \(\frac{7}{9} - \frac{2}{3}\)  
76. \(\frac{3}{4} - \frac{1}{6}\)

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