Solving Equations with the Variable on Each Side

What You'll Learn

- Solve equations with the variable on each side.
- Solve equations involving grouping symbols.

How can an equation be used to determine when two populations are equal?

In 1995, there were 18 million Internet users in North America. Of this total, 12 million were male, and 6 million were female. During the next five years, the number of male Internet users on average increased 7.6 million per year, and the number of female Internet users increased 8 million per year. If this trend continues, the following expressions represent the number of male and female Internet users \( x \) years after 1995.

Male Internet Users: \( 12 + 7.6x \)
Female Internet Users: \( 6 + 8x \)

The equation \( 12 + 7.6x = 6 + 8x \) represents the time at which the number of male and female Internet users are equal. Notice that this equation has the variable \( x \) on each side.

VARIABLES ON EACH SIDE  Many equations contain variables on each side. To solve these types of equations, first use the Addition or Subtraction Property of Equality to write an equivalent equation that has all of the variables on one side.

Example 1  Solve an Equation with Variables on Each Side

Solve \(-2 + 10p = 8p - 1\). Then check your solution.

\[
\begin{align*}
-2 + 10p &= 8p - 1 & \text{Original equation} \\
-2 + 10p - 8p &= 8p - 1 - 8p & \text{Subtract } 8p \text{ from each side.} \\
-2 + 2p &= -1 & \text{Simplify.} \\
-2 + 2p + 2 &= -1 + 2 & \text{Add 2 to each side.} \\
2p &= 1 & \text{Simplify.} \\
\frac{2p}{2} &= \frac{1}{2} & \text{Divide each side by 2.} \\
p &= \frac{1}{2} \text{ or } 0.5 & \text{Simplify.}
\end{align*}
\]

CHECK

\[
\begin{align*}
-2 + 10p &= 8p - 1 & \text{Original equation} \\
-2 + 10(0.5) &\neq 8(0.5) - 1 & \text{Substitute 0.5 for } p. \\
-2 + 5 &\neq 4 - 1 & \text{Multiply.} \\
3 &\neq 3 & \text{The solution is } \frac{1}{2} \text{ or } 0.5.
\end{align*}
\]
GROUPING SYMBOLS  When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example 2  Solve an Equation with Grouping Symbols

Solve $4(2r - 8) = \frac{1}{7}(49r + 70)$. Then check your solution.

\[
4(2r - 8) = \frac{1}{7}(49r + 70) \quad \text{Original equation}
\]

\[
8r - 32 = 7r + 10 \quad \text{Distributive Property}
\]

\[
8r - 32 - 7r = 7r + 10 - 7r \quad \text{Subtract 7r from each side.}
\]

\[
r - 32 = 10 \quad \text{Simplify.}
\]

\[
r - 32 + 32 = 10 + 32 \quad \text{Add 32 to each side.}
\]

\[
r = 42 \quad \text{Simplify.}
\]

CHECK

\[
4(2r - 8) = \frac{1}{7}(49r + 70) \quad \text{Original equation}
\]

\[
4[2(42) - 8] \neq \frac{1}{7}[49(42) + 70] \quad \text{Substitute 42 for r.}
\]

\[
4(84 - 8) \neq \frac{1}{7}(2058 + 70) \quad \text{Multiply.}
\]

\[
4(76) \neq \frac{1}{7}(2128) \quad \text{Add and subtract.}
\]

\[
304 \neq 304 \checkmark
\]

The solution is 42.

Some equations with the variable on each side may have no solution. That is, there is no value of the variable that will result in a true equation.

Example 3  No Solutions

Solve $2m + 5 = 5(m - 7) - 3m$.

\[
2m + 5 = 5(m - 7) - 3m \quad \text{Original equation}
\]

\[
2m + 5 = 5m - 35 - 3m \quad \text{Distributive Property}
\]

\[
2m + 5 = 2m - 35 \quad \text{Simplify.}
\]

\[
2m + 5 - 2m = 2m - 35 - 2m \quad \text{Subtract 2m from each side.}
\]

\[
5 = -35 \quad \text{This statement is false.}
\]

Since $5 = -35$ is a false statement, this equation has no solution.

An equation that is true for every value of the variable is called an identity.

Example 4  An Identity

Solve $3(r + 1) - 5 = 3r - 2$.

\[
3(r + 1) - 5 = 3r - 2 \quad \text{Original equation}
\]

\[
3r + 3 - 5 = 3r - 2 \quad \text{Distributive Property}
\]

\[
3r - 2 = 3r - 2 \quad \text{Reflexive Property of Equality}
\]

Since the expressions on each side of the equation are the same, this equation is an identity. The statement $3(r + 1) - 5 = 3r - 2$ is true for all values of $r$. 
Concept Summary

Steps for Solving Equations

Step 1 Use the Distributive Property to remove the grouping symbols.
Step 2 Simplify the expressions on each side of the equals sign.
Step 3 Use the Addition and/or Subtraction Properties of Equality to get the variables on one side of the equals sign and the numbers without variables on the other side of the equals sign.
Step 4 Simplify the expressions on each side of the equals sign.
Step 5 Use the Multiplication or Division Property of Equality to solve.
- If the solution results in a false statement, there is no solution of the equation.
- If the solution results in an identity, the solution is all numbers.

Example 5 Use Substitution to Solve an Equation

Multiple-Choice Test Item

Solve $2(b - 3) + 5 = 3(b - 1)$.

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<thead>
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<tr>
<td>A</td>
<td>-2</td>
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<td>D</td>
<td>3</td>
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Read the Test Item
You are asked to solve an equation.

Solve the Test Item
You can solve the equation or substitute each value into the equation and see if it makes the equation true. We will solve by substitution.

A $2(b - 3) + 5 = 3(b - 1)$
B $2(b - 3) + 5 = 3(b - 1)$
2(-2 - 3) + 5 $\neq$ 3(-2 - 1)
2(-5) + 5 $\neq$ 3(-3)
-10 + 5 $\neq$ -9
$-5 \neq -9$

B $2(b - 3) + 5 = 3(b - 1)$
2(-1) + 5 $\neq$ 3(1)
-2 + 5 $\neq$ 3
3 = 3 $\checkmark$

Since the value 2 results in a true statement, you do not need to check -3 and 3.
The answer is B.

Check for Understanding

Concept Check 1. Determine whether each solution is correct. If the solution is not correct, find the error and give the correct solution.

a. $2(g + 5) = 22$
   $2g + 5 = 22$
   $2g + 5 - 5 = 22 - 5$
   $2g = 17$
   $\frac{2g}{2} = 17$
   $g = 8.5$

b. $5d = 2d - 18$
   $5d - 2d = 2d - 18 - 2d$
   $3d = -18$
   $\frac{3d}{3} = -18$
   $d = -6$

c. $-6z + 13 = 7z$
   $-6z + 13 - 7z = 7z - 6z$
   $-13z + 13 = 0$
   $-13z = -13$
   $\frac{-13z}{-13} = \frac{-13}{-13}$
   $z = 1$

www.algebra1.com/extra_examples
2. Explain how to determine whether an equation is an identity.
3. OPEN ENDED Find a counterexample to the statement all equations have a solution.

**Guided Practice**

4. Justify each step.

\[ 6n + 7 = 8n - 13 \]
\[ 6n + 7 - 6n = 8n - 13 - 6n \]
\[ 7 = 2n - 13 \]
\[ 7 + 13 = 2n - 13 + 13 \]
\[ 20 = 2n \]
\[ \frac{20}{2} = \frac{2n}{2} \]
\[ 10 = n \]

5. Solve each equation. Then check your solution.

5. \[ 20c + 5 = 5c + 65 \]
6. \[ \frac{3}{8} - \frac{1}{4}t = \frac{1}{2}t - \frac{3}{4} \]
7. \[ 3(a - 5) = -6 \]
8. \[ 7 - 3r = r - 4(2 + r) \]
9. \[ 6 = 3 + 5(d - 2) \]
10. \[ \frac{c + 1}{8} = \frac{c}{4} \]
11. \[ 5h - 7 = 5(h - 2) + 3 \]
12. \[ 5.4w + 8.2 = 9.8w - 2.8 \]

**Standardized Test Practice**

13. Solve \[ 75 - 9t = 5(-4 + 2t) \].

A) -5  B) -4  C) 4  D) 5

**Practice and Apply**


\[ \frac{3m - 2}{5} = \frac{7}{10} \]
\[ \frac{3m - 2}{5} = \frac{7}{10} \]
\[ (3m - 2)\frac{2}{5} = 7 \]
\[ 6m - 4 = 7 \]
\[ 6m - 4 + 4 = 7 + 4 \]
\[ 6m = 11 \]
\[ \frac{6m}{6} = \frac{11}{6} \]
\[ m = \frac{11}{6} \]

15. \[ v + 9 = 7v + 9 \]

\[ v + 9 - v = 7v + 9 - v \]

\[ v + 9 - v = 7v + 9 - v \]

\[ 9 = 6v + 9 \]
\[ 9 - 9 = 6v + 9 - 9 \]
\[ 0 = 6v \]
\[ 0 = \frac{6v}{6} \]
\[ 0 = v \]

16. \[ 3 - 4q = 10q + 10 \]
17. \[ 3k - 5 = 7k - 21 \]
18. \[ 5t - 9 = -3t + 7 \]
19. \[ 8s + 9 = 7s + 6 \]
20. \[ \frac{3}{4}n + 16 = 2 - \frac{1}{8}n \]
21. \[ \frac{1}{4} - \frac{2}{3}y = \frac{3}{4} - \frac{1}{3}y \]
22. \[ 8 = 4(3c + 5) \]
23. \[ 7(m - 3) = 7 \]
24. \[ 6(r + 2) - 4 = -10 \]
25. \[ 5 - \frac{1}{2}(x - 6) = 4 \]
26. \[ 4(2a - 1) = -10(a - 5) \]
27. \[ 4(f - 2) = 4f \]
28. \[ 3(1 + d) - 5 = 3d - 2 \]
29. \[ 2(w - 3) + 5 = 3(w - 1) \]
30. \( \frac{3}{2}y - y = 4 + \frac{1}{2}y \)
31. \( 3 + \frac{2}{5}b = 11 - \frac{2}{5}b \)
32. \( \frac{1}{4}(7 + 3x) = -\frac{3}{8} \)
33. \( \frac{1}{6}(a - 4) = \frac{1}{3}(2a + 4) \)
34. \( 28 - 2.2x = 11.6x + 262.6 \)
35. \( 1.03p - 4 = -2.15p + 8.72 \)
36. \( 18 - 3.8t = 7.36 - 1.9t \)
37. \( 13.7v - 6.5 = -2.3v + 8.3 \)
38. \( 2(s + 3(s - 1)) = 18 \)
39. \( -3(2n - 5) = 0.5(-12n + 30) \)

40. One half of a number increased by 16 is four less than two thirds of the number. Find the number.

41. The sum of one half of a number and 6 equals one third of the number. What is the number?

42. **NUMBER THEORY** Twice the greater of two consecutive odd integers is 13 less than three times the lesser number. Find the integers.

43. **NUMBER THEORY** Three times the greatest of three consecutive even integers exceeds twice the least by 38. What are the integers?

44. **HEALTH** When exercising, a person's pulse rate should not exceed a certain limit, which depends on his or her age. This maximum rate is represented by the expression \( 0.8(220 - a) \), where \( a \) is age in years. Find the age of a person whose maximum pulse is 152.

45. **HARDWARE** Traditionally, nails are given names such as 2-penny, 3-penny, and so on. These names describe the lengths of the nails. What is the name of a nail that is 2\( \frac{1}{2} \) inches long?

\[ x \text{-penny nail} \]

\[ \text{nail length} = 1 + \frac{1}{4}(x - 2) \]

Source: *World Book Encyclopedia*

46. **TECHNOLOGY** About 4.9 million households had one brand of personal computers in 2001. The use of these computers grew at an average rate of 0.275 million households a year. In 2001, about 2.5 million households used another type of computer. The use of these computers grew at an average rate of 0.7 million households a year. How long will it take for the two types of computers to be in the same number of households?

47. **GEOMETRY** The rectangle and square shown below have the same perimeter. Find the dimensions of each figure.

\[ 3x + 1 \]

\[ x \]

\[ 3x \]

48. **ENERGY** Use the information on energy at the left. The amount of energy \( E \) in BTUs needed to raise the temperature of water is represented by the equation \( E = \frac{w(t_f - t_o)}{} \). In this equation, \( w \) represents the weight of the water in pounds, \( t_f \) represents the final temperature in degrees Fahrenheit, and \( t_o \) represents the original temperature in degrees Fahrenheit. A 50-gallon water heater is 60% efficient. If 10 cubic feet of natural gas are used to raise the temperature of water with the original temperature of 50°F, what is the final temperature of the water? (One gallon of water weighs about 8 pounds.)

49. **CRITICAL THINKING** Write an equation that has one or more grouping symbols, the variable on each side of the equals sign, and a solution of \(-2\).
50. **Writing in Math** Answer the question that was posed at the beginning of the lesson.

How can an equation be used to determine when two populations are equal?

Include the following in your answer:
- a list of the steps needed to solve the equation,
- the year when the number of female Internet users will equal the number of male Internet users according to the model, and
- an explanation of why this method can be used to predict future events.

51. Solve $8x - 3 = 5(2x + 1)$.
   - A 4
   - B 2
   - C -2
   - D -4

52. Solve $5n + 4 = 7(n + 1) - 2n$.
   - A 0
   - B -1
   - C no solution
   - D all numbers

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**Maintain Your Skills**

**Mixed Review** Solve each equation. Then check your solution.  (Lesson 3-4)

53. $\frac{2}{9}v - 6 = 14$
54. $\frac{x - 3}{7} = -2$
55. $5 - 9w = 23$

**Health** For Exercises 56 and 57, use the following information.

Ebony burns 4.5 Calories per minute pushing a lawn mower.  (Lesson 3-3)

56. Write a multiplication equation representing the number of Calories C burned by Ebony if she pushes the lawn mower for $m$ minutes.

57. How long will it take Ebony to burn 150 Calories mowing the lawn?

**Use each set of data to make a line plot.** (Lesson 2-5)

58. 13, 15, 11, 15, 16, 17, 12, 12, 13, 15, 16, 15
59. 22, 25, 19, 21, 22, 24, 22, 25, 28, 21, 24, 22

**Find each sum or difference.** (Lesson 2-2)

60. $-10 + (-17)$
61. $-12 - (-8)$
62. $6 - 14$

**Write a counterexample for each statement.** (Lesson 1-7)

63. If the sum of two numbers is even, then both addends are even.
64. If you are baking cookies, you will need chocolate chips.

**Evaluate each expression when $a = 5$, $b = 8$, $c = 7$, $x = 2$, and $y = 1$.** (Lesson 1-7)

65. $\frac{3a^2}{b + c}$
66. $x(a + 2b) - y$
67. $5(x + 2y) - 4a$

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**Getting Ready for the Next Lesson**

**Prerequisite Skill** Simplify each fraction.

(To review simplifying fractions, see pages 798 and 799.)

68. $\frac{12}{15}$
69. $\frac{28}{49}$
70. $\frac{36}{60}$
71. $\frac{8}{120}$

72. $\frac{108}{9}$
73. $\frac{28}{42}$
74. $\frac{16}{40}$
75. $\frac{19}{57}$