**What You’ll Learn**

- Solve mixture problems.
- Solve uniform motion problems.

**Vocabulary**

- weighted average
- mixture problem
- uniform motion problem

**How are scores calculated in a figure skating competition?**

In individual figure skating competitions, the score for the long program is worth twice the score for the short program. Suppose Olympic gold medal winner Ilia Kulik scores 5.5 in the short program and 5.8 in the long program at a competition. His final score is determined using a weighted average.

\[
\frac{5.5(1) + 5.8(2)}{1 + 2} = \frac{5.5 + 11.6}{3} = \frac{17.1}{3} \approx 5.7
\]

His final score would be 5.7.

**MIXTURE PROBLEMS** Ilia Kulik’s average score is an example of a weighted average. The **weighted average** of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units.

Mixture problems are problems in which two or more parts are combined into a whole. They are solved using weighted averages.

**Example 1** Solve a Mixture Problem with Prices

**TRAIL MIX** Assorted dried fruit sells for $5.50 per pound. How many pounds of mixed nuts selling for $4.75 per pound should be mixed with 10 pounds of dried fruit to obtain a trail mix that sells for $4.95 per pound?

Let \( w \) = the number of pounds of mixed nuts in the mixture. Make a table.

<table>
<thead>
<tr>
<th>Units (lb)</th>
<th>Price per Unit (lb)</th>
<th>Total Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dried Fruit</td>
<td>10</td>
<td>$5.50</td>
</tr>
<tr>
<td>Mixed Nuts</td>
<td>( w )</td>
<td>$4.75</td>
</tr>
<tr>
<td>Trail Mix</td>
<td>( 10 + w )</td>
<td>$4.95</td>
</tr>
</tbody>
</table>

\[
\text{Price of dried fruit} + \frac{\text{price of nuts}}{4.75w} = \frac{\text{price of trail mix}}{4.95(10 + w)}
\]

\[
5.50(10) + 4.75w = 4.95(10 + w)
\]

Original equation

\[
55.00 + 4.75w = 49.50 + 4.95w
\]

Distributive Property

\[
55.00 + 4.75w - 4.75w = 49.50 + 4.95w - 4.75w
\]

Subtract 4.75\( w \) from each side.

\[
55.00 = 49.50 + 0.20w
\]

Simplify.

\[
55.00 - 49.50 = 49.50 + 0.20w - 49.50
\]

Subtract 49.50 from each side.

\[
5.50 = 0.20w
\]

Simplify.

\[
\frac{5.50}{0.20} = \frac{0.20w}{0.20}
\]

Divide each side by 0.20.

\[
27.5 = w
\]

Simplify.

27.5 pounds of nuts should be mixed with 10 pounds of dried fruit.
Sometimes mixture problems are expressed in terms of percents.

**Example 2** Solve a Mixture Problem with Percents

**SCIENCE** A chemistry experiment calls for a 30% solution of copper sulfate. Kendra has 40 milliliters of 25% solution. How many milliliters of 60% solution should she add to obtain the required 30% solution?

Let \( x \) = the amount of 60% solution to be added. Make a table.

<table>
<thead>
<tr>
<th>Amount of Solution (mL)</th>
<th>Amount of Copper Sulfate</th>
</tr>
</thead>
<tbody>
<tr>
<td>25% Solution</td>
<td>40</td>
</tr>
<tr>
<td>60% Solution</td>
<td>( x )</td>
</tr>
<tr>
<td>30% Solution</td>
<td>40 + ( x )</td>
</tr>
</tbody>
</table>

Write and solve an equation using the information in the table.

\[
\frac{0.25(40)}{25\% \text{ solution}} + \frac{0.60x}{60\% \text{ solution}} = \frac{0.30(40 + x)}{30\% \text{ solution}}
\]

\[
0.25(40) + 0.60x = 0.30(40 + x)
\]

Original equation

Distributive Property

\[
10 + 0.60x = 12 + 0.30x
\]

Subtract 0.30\( x \) from each side.

\[
10 + 0.60x - 0.30x = 12
\]

Simplify.

\[
10 + 0.30x = 12
\]

Subtract 10 from each side.

\[
0.30x = 2
\]

Simplify.

\[
\frac{0.30x}{0.30} = \frac{2}{0.30}
\]

Divide each side by 0.30.

\[
x = 6.67
\]

Simplify.

Kendra should add 6.67 milliliters of the 60% solution to the 40 milliliters of the 25% solution.

**UNIFORM MOTION PROBLEMS** Motion problems are another application of weighted averages. Uniform motion problems are problems where an object moves at a certain speed, or rate. The formula \( d = rt \) is used to solve these problems. In the formula, \( d \) represents distance, \( r \) represents rate, and \( t \) represents time.

**Example 3** Solve for Average Speed

**TRAVEL** On Alberto’s drive to his aunt’s house, the traffic was light, and he drove the 45-mile trip in one hour. However, the return trip took him two hours. What was his average speed for the round trip?

To find the average speed for each leg of the trip, rewrite \( d = rt \) as \( r = \frac{d}{t} \).

**Going**

\[
r = \frac{d}{t} = \frac{45 \text{ miles}}{1 \text{ hour}} = 45 \text{ miles per hour}
\]

**Returning**

\[
r = \frac{d}{t} = \frac{45 \text{ miles}}{2 \text{ hours}} = 22.5 \text{ miles per hour}
\]
You may think that the average speed of the trip would be \( \frac{45 + 22.5}{2} \) or 33.75 miles per hour. However, Alberto did not drive at these speeds for equal amounts of time. You must find the weighted average for the trip.

**Round Trip**

\[
M = \frac{45(1) + 22.5(2)}{1 + 2} \quad \text{Definition of weighted average}
\]

\[
= \frac{90}{3} \quad \text{or} \quad 30 \quad \text{Simplify.}
\]

Alberto's average speed was 30 miles per hour.

Sometimes a table is useful in solving uniform motion problems.

**Example 4 Solve a Problem Involving Speeds of Two Vehicles**

**SAFETY** Use the information about sirens at the left. A car and an emergency vehicle are heading toward each other. The car is traveling at a speed of 30 miles per hour or about 44 feet per second. The emergency vehicle is traveling at a speed of 50 miles per hour or about 74 feet per second. If the vehicles are 1000 feet apart and the conditions are ideal, in how many seconds will the driver of the car first hear the siren?

Draw a diagram. The driver can hear the siren when the total distance traveled by the two vehicles equals 1000 - 440 or 560 feet.

Let \( t \) = the number of seconds until the driver can hear the siren.

Make a table of the information.

<table>
<thead>
<tr>
<th></th>
<th>( r )</th>
<th>( t )</th>
<th>( d = rt )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car</td>
<td>44</td>
<td>( t )</td>
<td>44( t )</td>
</tr>
<tr>
<td>Squad</td>
<td>74</td>
<td>( t )</td>
<td>74( t )</td>
</tr>
</tbody>
</table>

Write an equation.

\[
\text{Distance traveled by car} + \text{distance traveled by emergency vehicle} = 560
\]

\[
44t + 74t = 560 \quad \text{Original equation}
\]

\[
118t = 560 \quad \text{Simplify.}
\]

\[
\frac{118t}{118} = \frac{560}{118} \quad \text{Divide each side by 118.}
\]

\[
t = 4.75 \quad \text{Round to the nearest hundredth.}
\]

The driver of the car will hear the siren in about 4.75 seconds.
Check for Understanding

Concept Check
1. OPEN ENDED Give a real-world example of a weighted average.
2. Write the formula used to solve uniform motion problems and tell what each letter represents.
3. Make a table that can be used to solve the following problem.
Lakeisha has $2.55 in dimes and quarters. She has 8 more dimes than quarters. How many quarters does she have?

Guided Practice
FOOD For Exercises 4–7, use the following information.
How many quarts of pure orange juice should Michael add to a 10% orange drink to create 6 quarts of a 40% orange juice mixture? Let \( p \) represent the number of quarts of pure orange juice he should add to the orange drink.
4. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th>Quarts</th>
<th>Amount of Orange Juice</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% Juice</td>
<td>( 6 - p )</td>
</tr>
<tr>
<td>100% Juice</td>
<td>( p )</td>
</tr>
<tr>
<td>40% Juice</td>
<td></td>
</tr>
</tbody>
</table>

5. Write an equation to represent the problem.
6. How much pure orange juice should Michael use?
7. How much 10% juice should Michael use?
8. BUSINESS The Nut Shoppe sells walnuts for $4.00 a pound and cashews for $7.00 a pound. How many pounds of cashews should be mixed with 10 pounds of walnuts to obtain a mixture that sells for $5.50 a pound?
9. GRADES Many schools base a student’s grade point average, or GPA, on the student’s grade and the class credit rating. Brittany’s grade card for this semester is shown. Find Brittany’s GPA if a grade of A equals 4 and a B equals 3.

<table>
<thead>
<tr>
<th>Grade Card</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
</tr>
<tr>
<td>Credit</td>
</tr>
<tr>
<td>Rating</td>
</tr>
<tr>
<td>Grade</td>
</tr>
<tr>
<td>Algebra 1</td>
</tr>
<tr>
<td>Science</td>
</tr>
<tr>
<td>English</td>
</tr>
<tr>
<td>Spanish</td>
</tr>
<tr>
<td>Phys. Ed.</td>
</tr>
</tbody>
</table>

10. CYCLING Two cyclists begin traveling in the same direction on the same bike path. One travels at 20 miles per hour, and the other travels at 14 miles per hour. When will the cyclists be 15 miles apart?

Practice and Apply

Homework Help
For Exercises See Examples
11–18, 22–25, 1, 2
27–29, 33
19–21
26, 30–32, 34
3, 4

Extra Practice
See page 828.

BUSINESS For Exercises 11–14, use the following information.
Cookies Inc. sells peanut butter cookies for $6.50 per dozen and chocolate chip cookies for $9.00 per dozen. Yesterday, they sold 85 dozen more peanut butter cookies than chocolate chip cookies. The total sales for both types of cookies were $4055.50. Let \( p \) represent the number of dozens of peanut butter cookies sold.

11. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th>Number of Dozens</th>
<th>Price per Dozen</th>
<th>Total Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peanut Butter Cookies</td>
<td>( p )</td>
<td></td>
</tr>
<tr>
<td>Chocolate Chip Cookies</td>
<td>( p - 85 )</td>
<td></td>
</tr>
</tbody>
</table>

12. Write an equation to represent the problem.
13. How many dozen peanut butter cookies were sold?
14. How many dozen chocolate chip cookies were sold?
**METALS** For Exercises 15–18, use the following information.

In 2000, the international price of gold was $270 per ounce, and the international price of silver was $5 per ounce. Suppose gold and silver were mixed to obtain 15 ounces of an alloy worth $164 per ounce. Let \( g \) represent the amount of gold used in the alloy.

15. Copy and complete the table representing the problem.

<table>
<thead>
<tr>
<th>Number of Ounces</th>
<th>Price per Ounce</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>( g )</td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>15 – ( g )</td>
<td></td>
</tr>
<tr>
<td>Alloy</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

16. Write an equation to represent the problem.

17. How much gold was used in the alloy?

18. How much silver was used in the alloy?

**TRAVEL** For Exercises 19–21, use the following information.

Two trains leave Pittsburgh at the same time, one traveling east and the other traveling west. The eastbound train travels at 40 miles per hour, and the westbound train travels at 30 miles per hour. Let \( t \) represent the amount of time since their departure.

19. Copy and complete the table representing the situation.

<table>
<thead>
<tr>
<th>( r )</th>
<th>( t )</th>
<th>( d = rt )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastbound Train</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Westbound Train</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

20. Write an equation that could be used to determine when the trains will be 245 miles apart.

21. In how many hours will the trains be 245 miles apart?

**FUND-RAISING** The Madison High School marching band sold gift wrap. The gift wrap in solid colors sold for $4.00 per roll, and the print gift wrap sold for $6.00 per roll. The total number of rolls sold was 480, and the total amount of money collected was $2340. How many rolls of each kind of gift wrap were sold?

22. **COFFEE** Charley Baroni owns a specialty coffee store. He wants to create a special mix using two coffees, one priced at $6.40 per pound and the other priced at $7.28 per pound. How many pounds of the $7.28 coffee should he mix with 9 pounds of the $6.40 coffee to sell the mixture for $6.95 per pound?

23. **FOOD** Refer to the graphic at the right. How much whipping cream and 2% milk should be mixed to obtain 35 gallons of milk with 4% butterfat?

24. **METALS** An alloy of metals is 25% copper. Another alloy is 50% copper. How much of each alloy should be used to make 1000 grams of an alloy that is 45% copper?

25. **TRAVEL** An airplane flies 1000 miles due east in 2 hours and 1000 miles due south in 3 hours. What is the average speed of the airplane?
27. SCIENCE Hector is performing a chemistry experiment that requires 140 milliliters of a 30% copper sulfate solution. He has a 25% copper sulfate solution and a 60% copper sulfate solution. How many milliliters of each solution should he mix to obtain the needed solution?

28. CAR MAINTENANCE One type of antifreeze is 40% glycol, and another type of antifreeze is 60% glycol. How much of each kind should be used to make 100 gallons of antifreeze that is 48% glycol?

29. GRADES In Ms. Martinez's science class, a test is worth three times as much as a quiz. If a student has test grades of 85 and 92 and quiz grades of 82, 75, and 95, what is the student's average grade?

30. RESCUE A fishing trawler has radioed the Coast Guard for a helicopter to pick up an injured crew member. At the time of the emergency message, the trawler is 660 kilometers from the helicopter and heading toward it. The average speed of the trawler is 30 kilometers per hour, and the average speed of the helicopter is 300 kilometers per hour. How long will it take the helicopter to reach the trawler?

31. ANIMALS A cheetah is 300 feet from its prey. It starts to sprint toward its prey at 90 feet per second. At the same time, the prey starts to sprint at 70 feet per second. When will the cheetah catch its prey?

32. TRACK AND FIELD A sprinter has a bad start, and his opponent is able to start 1 second before him. If the sprinter averages 8.2 meters per second and his opponent averages 8 meters per second, will he be able to catch his opponent before the end of the 200-meter race? Explain.

33. CAR MAINTENANCE A car radiator has a capacity of 16 quarts and is filled with a 25% antifreeze solution. How much must be drained off and replaced with pure antifreeze to obtain a 40% antifreeze solution?

34. TRAVEL An express train travels 80 kilometers per hour from Ironton to Wildwood. A local train, traveling at 48 kilometers per hour, takes 2 hours longer for the same trip. How far apart are Ironton and Wildwood?

35. FOOTBALL NFL quarterbacks are rated for their passing performance by a type of weighted average as described in the formula below.

\[ R = \frac{[50 + 2000(C \div A) + 8000(T \div A) - 10,000(I \div A) + 100(Y \div A)]}{24} \]

In this formula,
\begin{itemize}
  \item \( R \) represents the rating,
  \item \( C \) represents number of completions,
  \item \( A \) represents the number of passing attempts,
  \item \( T \) represents the number of touchdown passes,
  \item \( I \) represents the number of interceptions, and
  \item \( Y \) represents the number of yards gained by passing.
\end{itemize}

In the 2000 season, Daunte Culpepper had 297 completions, 474 passing attempts, 33 touchdown passes, 16 interceptions, and 3937 passing yards. What was his rating for that year?

Online Research Data Update What is the current passing rating for your favorite quarterback? Visit www.algebra1.com/data_update to get statistics on quarterbacks.

36. CRITICAL THINKING Write a mixture problem for the equation \( 1.00x + 0.28(40) = 0.40(x + 40) \).
37. **Writing in Math**  
Answer the question that was posed at the beginning of the lesson.

**How are scores calculated in a figure skating competition?**

Include the following in your answer:

- an explanation of how a weighted average can be used to find a skating score, and
- a demonstration of how to find the weighted average of a skater who received a 4.9 in the short program and a 5.2 in the long program.

38. Eula Jones is investing $6000 in two accounts, part at 4.5% and the remainder at 6%. If \( d \) represents the number of dollars invested at 4.5%, which expression represents the amount of interest earned in one year by the account paying 6%?

\[\begin{align*}
\text{A} & \quad 0.06d \\
\text{B} & \quad 0.06(d - 6000) \\
\text{C} & \quad 0.06(d + 6000) \\
\text{D} & \quad 0.06(6000 - d)
\end{align*}\]

39. Todd drove from Boston to Cleveland, a distance of 616 miles. His breaks, gasoline, and food stops took 2 hours. If his trip took 16 hours altogether, what was his average speed?

\[\begin{align*}
\text{A} & \quad 38.5 \text{ mph} \\
\text{B} & \quad 40 \text{ mph} \\
\text{C} & \quad 44 \text{ mph} \\
\text{D} & \quad 47.5 \text{ mph}
\end{align*}\]

Maintain Your Skills

**Mixed Review**  
Solve each equation for the variable specified.  
(Lesson 3-8)

40. \(3t - 4 = 6t - s\), for \(t\)

41. \(a + 6 = \frac{b - 1}{4}\), for \(b\)

State whether each percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent.  
(Lesson 3-7)

42. original: 25  
   new: 14

43. original: 35  
   new: 42

44. original: 244  
   new: 300

45. If the probability that an event will occur is \(\frac{2}{3}\), what are the odds that the event will occur?  
(Lesson 2-6)

46. \((2b)(-3a)\)

47. \(3x(-3y) + (-6x)(-2y)\)

48. \(5s(-6t) + 2s(-8t)\)

Simplify each expression.  
(Lesson 2-3)

49. \[\begin{align*}
-2 & \quad -1 & \quad 0 & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 & \quad 6 & \quad 7
\end{align*}\]

50. \[\begin{align*}
-1 & \quad 0 & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 & \quad 6 & \quad 7 & \quad 8
\end{align*}\]

**WebQuest Internet Project**

**Can You Fit 100 Candles on a Cake?**

It’s time to complete your project. Use the information and data you have gathered about living to be 100 to prepare a portfolio or Web page. Be sure to include graphs and/or tables in the presentation.

www.algebra1.com/webquest