6-6
Graphing Inequalities in Two Variables

What You'll Learn
- Graph inequalities on the coordinate plane.
- Solve real-world problems involving linear inequalities.

How are inequalities used in budgets?
Hannah allots up to $30 a month for lunch on school days. On most days, she brings her lunch. She can also buy lunch at the cafeteria or at a fast-food restaurant. She spends an average of $3 a day at the cafeteria and an average of $4 a day at a restaurant. How many times a month can Hannah buy her lunch and remain within her budget?

Let \( x \) represent the number of days she buys lunch at the cafeteria, and let \( y \) represent the number of days she buys lunch at a restaurant. Then the following inequality can be used to represent the situation.

\[
3x + 4y \leq 30
\]

There are many solutions of this inequality.

GRAPH LINEAR INEQUALITIES
Like a linear equation in two variables, the solution set of an inequality in two variables is graphed on a coordinate plane. The solution set of an inequality in two variables is the set of all ordered pairs that satisfy the inequality.

Example 1 Ordered Pairs that Satisfy an Inequality

From the set \( \{(1, 6), (3, 0), (2, 2), (4, 3)\} \), which ordered pairs are part of the solution set for \( 3x + 2y < 12 \)?

Use a table to substitute the \( x \) and \( y \) values of each ordered pair into the inequality.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( 3x + 2y )</th>
<th>True or False</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>3(1) + 2(6)</td>
<td>15 &lt; 12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15 &lt; 12</td>
<td>false</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3(3) + 2(0)</td>
<td>9 &lt; 12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9 &lt; 12</td>
<td>true</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3(2) + 2(2)</td>
<td>10 &lt; 12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 &lt; 12</td>
<td>true</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3(4) + 2(3)</td>
<td>18 &lt; 12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18 &lt; 12</td>
<td>false</td>
</tr>
</tbody>
</table>

The ordered pairs \( (3, 0), (2, 2) \) are part of the solution set of \( 3x + 2y < 12 \). In the graph, notice the location of the two ordered pairs that are solutions for \( 3x + 2y < 12 \) in relation to the line.
The solution set for an inequality in two variables contains many ordered pairs when the domain and range are the set of real numbers. The graphs of all of these ordered pairs fill a region on the coordinate plane called a half-plane. An equation defines the boundary or edge for each half-plane.

**Key Concept**

- **Words** Any line in the plane divides the plane into two regions called half-planes. The line is called the boundary of each of the two half-planes.

- **Model**

Consider the graph of \( y > 4 \). First determine the boundary by graphing \( y = 4 \), the equation you obtain by replacing the inequality sign with an equals sign. Since the inequality involves \( y \)-values greater than 4, but not equal to 4, the line should be dashed. The boundary divides the coordinate plane into two half-planes.

To determine which half-plane contains the solution, choose a point from each half-plane and test it in the inequality.

Try \((3, 0)\). \n\[
\begin{align*}
  y &> 4 \\
  y &= 0 \\
  0 &> 4 \\
  \text{false}
\end{align*}
\]

Try \((5, 6)\). \n\[
\begin{align*}
  y &> 4 \\
  y &= 6 \\
  6 &> 4 \\
  \text{true}
\end{align*}
\]

The half-plane that contains \((5, 6)\) contains the solution. Shade that half-plane.

**Example 2** Graph an Inequality

Graph \( y - 2x \leq -4 \).

**Step 1** Solve for \( y \) in terms of \( x \).
\[
\begin{align*}
  y - 2x &\leq -4 & \text{Original inequality} \\
  y - 2x + 2x &\leq -4 + 2x & \text{Add } 2x \text{ to each side.} \\
  y &\leq 2x - 4 & \text{Simplify.}
\end{align*}
\]

**Step 2** Graph \( y = 2x - 4 \). Since \( y \leq 2x - 4 \) means \( y < 2x - 4 \) or \( y = 2x - 4 \), the boundary is included in the solution set. The boundary should be drawn as a solid line.

(continued on the next page)
Study Tip

**Origin as the Test Point**
Use the origin as a standard test point because the values are easy to substitute into the inequality.

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Step 3 Select a point in one of the half-planes and test it. Let's use \((0, 0)\).

\[
\begin{align*}
  y & \leq -2x - 4 \quad & \text{Original inequality} \\
  0 & \leq 2(0) - 4 \quad x = 0, y = 0 \\
  0 & \leq -4 \quad \text{false}
\end{align*}
\]

Since the statement is false, the half-plane containing the origin is not part of the solution. Shade the other half-plane.

**CHECK** Test a point in the other half-plane, for example, \((3, -3)\).

\[
\begin{align*}
  y & \leq -2x - 4 \quad & \text{Original inequality} \\
  -3 & \leq 2(3) - 4 \quad x = 3, y = -3 \\
  -3 & \leq 2 \quad \checkmark
\end{align*}
\]

Since the statement is true, the half-plane containing \((3, -3)\) should be shaded. Graph of the solution is correct.

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**SOLVE REAL-WORLD PROBLEMS** When solving real-world inequalities the domain and range of the inequality are often restricted to nonnegative numbers or whole numbers.

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**Example 3 Write and Solve an Inequality**

**Advertising** Rosa Padilla sells radio advertising in 30-second and 60-second time slots. During every hour, there are up to 15 minutes available for commercials. How many commercial slots can she sell for one hour of broadcasting?

**Step 1** Let \(x\) equal the number of 30-second commercials. Let \(y\) equal the number of 60-second or 1-minute commercials. Write an open sentence representing this situation.

\[
\frac{1}{2} \text{ min times the number of 30-s commercials plus the number of 1-min commercials is up to } 15 \text{ min}
\]

\[
\frac{1}{2} x + y \leq 15
\]

**Step 2** Solve for \(y\) in terms of \(x\).

\[
\begin{align*}
  \frac{1}{2} x + y & \leq 15 \quad \text{Original inequality} \\
  \frac{1}{2} x + y - \frac{1}{2} x & \leq 15 - \frac{1}{2} x \quad \text{Subtract } \frac{1}{2} x \text{ from both sides.} \\
  y & \leq 15 - \frac{1}{2} x \quad \text{Simplify.}
\end{align*}
\]

**Step 3** Since the open sentence includes the equation graph \(y = 15 - \frac{1}{2} x\) as a solid line. Test a point in one of the half-planes, for example \((0, 0)\). Shade the half-plane containing \((0, 0)\) since \(0 \leq 15 - \frac{1}{2}(0)\) is true.
Step 4  Examine the solution.
- Rosa cannot sell a negative number of commercials. Therefore, the domain and range contain only nonnegative numbers.
- She also cannot sell half of a commercial. Thus, only points in the shaded half-plane whose $x$- and $y$-coordinates are whole numbers are possible solutions.

One solution is $(12, 8)$. This represents twelve 30-second commercials and eight 60-second commercials in a one hour period.

**Check for Understanding**

**Concept Check**
1. Compare and contrast the graph of $y = x + 2$ and the graph of $y < x + 2$.
2. **OPEN ENDED** Write an inequality in two variables and graph it.
3. Explain why it is usually only necessary to test one point when graphing an inequality.

**Guided Practice**
Determine which ordered pairs are part of the solution set for each inequality.
4. $y \leq x + 1$, $\{(−1, 0), (3, 2), (2, 5), (−2, 1)\}$
5. $y > 2x$, $\{(2, 6), (0, −1), (3, 5), (−1, −2)\}$
6. Which graph represents $y − 2x \geq 2$?
   a.  
   b.  
   c.  

Graph each inequality.
7. $y \geq 4$
8. $y \leq 2x − 3$
9. $4 − 2x < −2$
10. $1 − y > x$

**Application**
11. **ENTERTAINMENT** Coach Riley wants to take her softball team out for pizza and soft drinks after the last game of the season. She doesn’t want to spend more than $60. Write an inequality that represents this situation and graph the solution set.
Determine which ordered pairs are part of the solution set for each inequality.

12. \( y \leq 3 - 2x \), \((0, 4), (-1, 3), (6, -8), (-4, 5)\)
13. \( y < 3x \), \((-3, 1), (-3, 2), (1, 1), (1, 2)\)
14. \( x + y < 11 \), \((5, 7), (-13, 10), (4, 4), (-6, -2)\)
15. \( 2x - 3y > 6 \), \((3, 2), (-2, -4), (6, 2), (5, 1)\)
16. \( 4y - 8 \geq 0 \), \((5, 1), (0, 2), (2, 5), (-2, 0)\)
17. \( 3x + 4y < 7 \), \((1, 1), (2, -1), (-1, 1), (-2, 4)\)
18. \( |x - 3| \leq y \), \((6, 4), (-1, 8), (-3, 2), (5, 7)\)
19. \( |y + 2| < x \), \((2, -4), (-1, -5), (6, -7), (0, 0)\)

Match each inequality with its graph.

20. \( 2y + x \leq 6 \) a. 

21. \( \frac{1}{2}x - y > 4 \) b. 

22. \( y > 3 + \frac{1}{2}x \) c. 

23. \( 4y + 2x \geq 16 \) d. 

24. Is the point \( A(2, 3) \) on, above, or below the graph of \( -2x + 3y = 5 \)?
25. Is the point \( B(0, 1) \) on, above, or below the graph of \( 4x - 3y = 4 \)?

Graph each inequality.

26. \( y < -3 \) 27. \( x \geq 2 \) 28. \( 5x + 10y > 0 \) 29. \( y < x \)
30. \( 2y - x \leq 6 \) 31. \( 6x + 3y > 9 \) 32. \( 3y - 4x \geq 12 \) 33. \( y \leq -2x - 4 \)
34. \( 8x - 6y < 10 \) 35. \( 3x - 1 \leq y \) 36. \( 3(x + 2y) > -18 \) 37. \( \frac{1}{2}(2x + y) < 2 \)

POSTAGE For Exercises 38 and 39, use the following information.
The U.S. Postal Service limits the size of packages to those in which the length of the longest side plus the distance around the thickest part is less than or equal to 108 inches.

38. Write an inequality that represents this situation.
39. Are there any restrictions on the domain or range?

Online Research Data Update What are the current postage rates and regulations? Visit www.algebra1.com/data_update to learn more.

SHIPPING For Exercises 40 and 41, use the following information.
A delivery truck is transporting televisions and microwaves to an appliance store. The weight limit for the truck is 4000 pounds. The televisions weigh 77 pounds, and the microwaves weigh 55 pounds.

40. Write an inequality for this situation.
41. Will the truck be able to deliver 35 televisions and 25 microwaves at once?
FALL DANCE  For Exercises 42–44, use the following information.
Tickets for the fall dance are $5 per person or $8 for couples. In order to cover
expenses, at least $1200 worth of tickets must be sold.
42. Write an inequality that represents this situation.
43. Graph the inequality.
44. If 100 single tickets and 125 couple tickets are sold, will the committee cover its
expenses?

45. CRITICAL THINKING  Graph the intersection of the graphs of $y \leq x - 1$ and
$y \geq -x$.
46. WRITING IN MATH  Answer the question that was posed at the beginning of the
lesson.
How are inequalities used in budgets?
Include the following in your answer:
• an explanation of the restrictions placed on the domain and range of the
inequality used to describe the number of times Hannah can buy her lunch, and
• three possible solutions of the inequality.

47. Which ordered pair is not a solution of $y - 2x < -5$?
A. (2, -2)  B. (-1, -8)  C. (4, 1)  D. (5, 6)

48. Which inequality is represented by the graph
at the right?
A. $2x + y < 1$  B. $2x + y > 1$
C. $2x + y \leq 1$  D. $2x + y \geq 1$

Standardized
Test Practice

Maintain Your Skills

Mixed Review  Solve each open sentence. Then graph the solution set.  \((Lesson 6-5)\)
49. $|3 + 2l| = 11$  50. $|x + 8| < 6$  51. $|2y + 5| \geq 3$

Solve each compound inequality. Then graph the solution.  \((Lesson 6-4)\)
52. $y + 6 > -1$ and $y - 2 < 4$  53. $m + 4 < 2$ or $m - 2 > 1$

State whether each percent of change is a percent of increase or decrease.
Then find the percent of change. Round to the nearest whole percent.  \((Lesson 3-7)\)
54. original: 200  55. original: 100  56. original: 53
new: 172  new: 142  new: 75

Solve each equation.  \((Lesson 3-4)\)
57. $\frac{d - 2}{3} = 7$  58. $3n + 6 = -15$  59. $35 + 20h = 100$

Simplify.  \((Lesson 2-4)\)
60. $-\frac{64}{4}$  61. $\frac{27c}{-9}$  62. $\frac{12a - 14b}{-2}$  63. $\frac{18y - 9}{3}$